*International Jl. on E-Learning (2006)* **5***(1),* 75-88

# Competence-Based Knowledge Structures for Personalised Learning

JÜRGEN HELLER, CHRISTINA STEINER, CORD HOCKEMEYER, AND DIETRICH ALBERT
*University of Graz - Graz, Austria*
juergen.heller@uni-graz.at
chr.steiner@uni-graz.at
cord.hockemeyer@uni-graz.at
dietrich.albert@uni-graz.at

Competence-based extensions of Knowledge Space Theory are suggested as a formal framework for implementing key features of personalised learning in technology-enhanced learning. The approach links learning objects and assessment problems to the relevant skills that are taught or required. Various ways to derive these skills from domain ontologies are discussed in detail. Moreover, it is shown that the approach induces structures on the assessment problems and learning objects, respectively, that can serve as a basis for an efficient adaptive assessment of the learners' skills, and for selecting personalised learning paths.

Personalised learning aims to tailor teaching to individual needs, interests, and aptitude to ensure that every learner achieves and reaches the highest standards possible. It usually proceeds by assessing the learner's current knowledge state and probably other individual characteristics or preferences, and by using the results of this assessment to inform further teaching. Knowledge Space Theory (Doignon & Falmagne, 1985, 1999; Falmagne, Koppen, Villano, Doignon, & Johannesen, 1990) provides a foundation for personalising the learning experience. The theory, in its original formalisation, is purely behaviouristic. Various approaches have been devised in order to theoretically explain the observed behaviour by considering underlying cognitive constructs (e.g. Falmagne et al., 1990). These approaches focus on items' difficulty components, their underlying demands, and skills or competencies, and processes for performing them.

The following section will give an introduction to the basic concepts of Knowledge Space Theory. Subsequently, an extension of Knowledge Space Theory is suggested as a formal framework that can serve as a basis for implementing personalised learning into a technology-enhanced learning system. This approach incorporates explicit reference to underlying skills and competencies and integrates learning objects into an originally behaviouristic formal psychological theory with its focus on knowledge assessment. Its discussion covers the derivation of skills and their structure from ontological information, and elaborates on the impact of skill assignments on both the assessment problems and the learning objects. It is shown that these assignments induce structures, which allow for designing efficient procedures for adaptive assessment of the learner's competencies, and for generating personalised learning paths.

## BASIC NOTIONS OF KNOWLEDGE SPACE THEORY

Knowledge Space Theory provides a set-theoretic framework for representing the knowledge of a learner in a certain domain, which is characterised by a set of assessment problems (subsequently denoted by $Q$). In this framework the knowledge state of an individual is identified with the set of problems the person is capable of solving. Due to mutual (psychological) dependencies between the problems not all potential knowledge states (i.e., subsets of problems) will actually be observed. If a correct solution to a certain problem can be inferred given another problem is mastered, then each knowledge state will contain the first problem whenever it contains the second one (i.e. the first problem may be considered a prerequisite to the second). To capture the relationships between the problems of a domain the notion of a surmise relation was introduced. Two problems $a$ and $b$ are in a surmise relation whenever from a correct solution to problem $b$ the mastery of problem $a$ can be surmised. A surmise relation can be illustrated by a so-called Hasse diagram (see Figure 1 for an example), where descending sequences of line segments indicate a surmise relation. According to the surmise relation shown in Figure 1, from a correct solution to problem $b$ the correct answer to problem $a$ can be surmised, while the mastery of problem $e$ implies correct answers to problems $a$, $b$, and $c$. A surmise relation restricts the number of possible knowledge states and forms a quasi-order on the set of assessment problems.

The collection of possible knowledge states of a given domain $Q$ is called a knowledge structure, whenever it contains the empty set Ø and the whole set $Q$. The knowledge structure $K$ induced by the surmise relation depicted in Figure 1 is given by

$$K = \{ \emptyset, \{a\}, \{c\}, \{a, c\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}, \{a, b, c, e\}, \{a, b, c, d\}, Q\}.$$

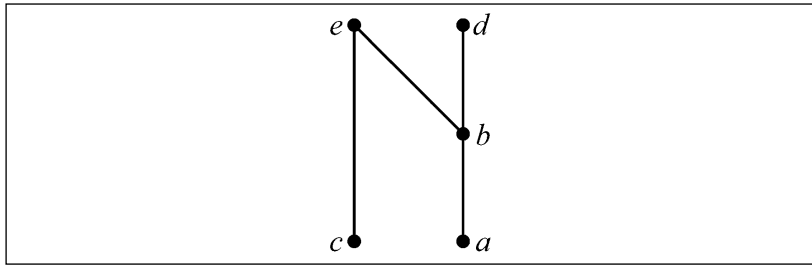


**Figure 1.** Example of a Hasse diagram illustrating a surmise relation on the knowledge domain $Q = \{a, b, c, d, e\}$

The possible knowledge states are naturally ordered by set-inclusion, which results in the diagram shown in Figure 2.

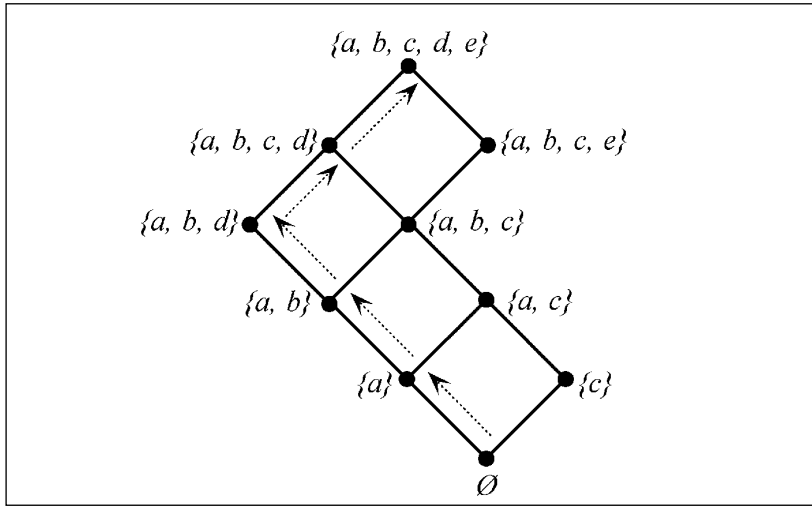


**Figure 2.** Knowledge structure $K$ induced by the surmise relation of Figure 1. The dashed arrows indicate a possible learning path.

Figure 2 illustrates that there are various possible learning paths for moving from the naive knowledge state (empty set Ø) to the knowledge state of full mastery (set $Q$). One of the possible learning paths is indicated by arrows describing the possible steps of a learning process. It suggests to initially present material related to problem $a$ (or, equivalently, $c$), followed by material related to problems $b$ or $c$ ($a$, respectively), and so on. Notice that the knowledge structure of Figure 2 is somehow special, as it allows for gradual learning. On the one hand, each knowledge state (except state $Q$) has at least one

immediate successor state that comprises all the same problems plus exactly one. On the other hand, each knowledge state (except state Ø) has at least one predecessor state that contains exactly the same problems, except one. A knowledge structure with these properties, in which learning can take place step by step, is called well-graded. According to Figure 2, for instance, the states $\{a, b, c, d\}$ and $\{a, b, c, e\}$ are the immediate successor states to the knowledge state $\{a, b, c\}$. The set $\{d, e\}$ constitutes the so-called *outer fringe* of the knowledge state $\{a, b, c\}$. It consists of exactly those problems that a learner having knowledge state $\{a, b, c\}$ should tackle next, and can thus form a basis for generating personalised learning paths. The knowledge state $\{a, b, c\}$ has also two predecessor states, which are $\{a, b\}$ and $\{a, c\}$. The set $\{b, c\}$ represents the so-called *inner fringe* of the knowledge state $\{a, b, c\}$. Its problems may be seen as corresponding to the most sophisticated content that has been learned recently. This is the content that the learner should revisit, when previously learned material is to be reviewed.

Besides providing the information relevant for generating personalised learning paths, a knowledge structure is at the core of an efficient adaptive procedure for knowledge assessment. It allows for uniquely determining the knowledge state by presenting the learner with only a subset of the problems (for more details see "Problem-Based Skill Assessment").

## COMPETENCE-BASED EXTENSIONS OF KNOWLEDGE SPACE THEORY

Although there is a commercial learning system that is based on Knowledge Space Theory, which is the ALEKS system (http://www.aleks.com), this approach suffers from its limitation to a purely behaviouristic perspective. In its original formalisation, Knowledge Space Theory focuses completely on the observable solution behaviour, and does not refer to both learning objects and skills or competencies that are to be taught. To overcome these limitations Knowledge Space Theory may be extended so that it incorporates explicit reference to learning objects and underlying skills and competencies. The subsequent considerations are based on previous work by Falmagne et al. (1990), Doignon (1994), Düntsch and Gediga (1995), Korossy (1997, 1999), Albert and Held (1994, 1999), Hockemeyer (2003), and Hockemeyer, Conlan, Wade, and Albert (2003). It not only integrates these different contributions, but also derives their implications for implementing a personalised learning system, and clarifies the role of domain ontologies.

Extended Knowledge Space Theory is dealing with three different sorts of entities, which are:

1. the set $Q$ of assessment problems,
2. the set $L$ of learning objects (LOs),
3. the set $S$ of skills relevant for solving the problems, and taught by the LOs.

Notice that the skills in the set $S$ are meant to provide a fine-grained, low-level description of the learner's capabilities. Usually, it is a whole bunch of skills that is tested by an assessment problem, or taught by a LO.

Each of these basic sets is assumed to be endowed with a structure, which we conceive as a collection of subsets of the respective set. In particular, we consider

- a knowledge structure on the set $Q$ of assessment problems,
- a learning structure on the set $L$ of LOs,
- a competence structure on the set of skills $S$.

As outlined, the knowledge structure constitutes the collection of possible knowledge states and forms the basis of the problem-based assessment of a student's competency (see "Problem-based Skill Assessment"). Usage of the notion "competency" in the present context is in line with the terminology of Doignon and Falmagne (1999), which refers to subsets of skills that are collected in the competence structure, and which may also be called competence states. A competence structure may either be explicitly established by identifying prerequisite relationships between skills (see "Deriving Skills and their Structure from Domain Ontologies") that restrict the set of possible competence states, or it may be indirectly induced by assigning skills to assessment problems (or LOs) (see "Assigning Skills to Assessment Problems" and "Assigning Skills to Learning Objects"). The learning structure together with a student's current competence state is used to generate a personalised learning path. Learning and competence structures are defined in complete analogy to the knowledge structure previously introduced. Now, the main goal is to identify the pieces of information that are needed for establishing those structures.

## SKILLS AND SKILL ASSIGNMENTS

### Deriving Skills and their Structure from Domain Ontologies

This section addresses the question of how to identify skills that are relevant and suitable for modelling the underlying constructs of assessment problems and learning object regarding a certain domain. As an alternative to cognitive task analysis (Korossy, 1999), querying experts (Zaluski, 2001), and systematic problem construction by applying the component-attribute approach (Albert & Held, 1994), we propose to utilise information coming from domain ontologies.

An ontology allows structuring a domain of knowledge with respect to its conceptual organization. It constitutes a specification of the concepts in a domain and the relations among them and thus, defines a common vocabulary of the knowledge domain. A common and natural way of representing ontologies is by concept maps. The ontological information provided by a

concept map can be used for identifying skills and for establishing a competence structure, respectively. In the sequel we outline two approaches, which differ with respect to the level of granularity of the underlying concept map.

***Identifying skills with substructures of a concept map.*** Skills in terms of competence-based Knowledge Space Theory may be identified with substructures of a concept map representing the ontological information of the respective domain. This actually assumes a quite fine-grained representation, as it is necessary for a detailed characterisation of learning content, for example. A specific skill that is required for solving problems, or that is taught by learning objects, can be identified with a subset of the propositions represented by the concept map. Consider, for instance, the knowledge domain of right triangles. Figure 3 illustrates a possible assessment problem from this domain.

Solving this geometry problem requires to know the Pythagorean Theorem and how to apply it. Knowing the Pythagorean Theorem may be assumed to constitute a skill, which corresponds to a substructure of a concept map. Figure 4 provides an exemplary concept map that highlights the substructure representing this skill. Note that not all the relevant skills can be constructued in this way. The ability of applying the Pythagorean Theorem, for example, may be regarded as a related, but separate skill, which has to be added to the set of considered skills.

The representation of skills in the concept map may also be used for deriving dependencies between skills, e.g. by set inclusion. If the representation of a skill $x$ in the concept map is a subset of that of a skill $y$, then skill $x$ constitutes a prerequisite to skill $y$.

***Using the component-attribute approach.*** Concept maps provide a tool for modelling the content of a knowledge domain, which is an essential part of curriculum and content analysis. Within this context the construction of concept maps aims at uncovering the prerequisite relations among the basic concepts within a topic, and between different topics of a subject. Such a con-

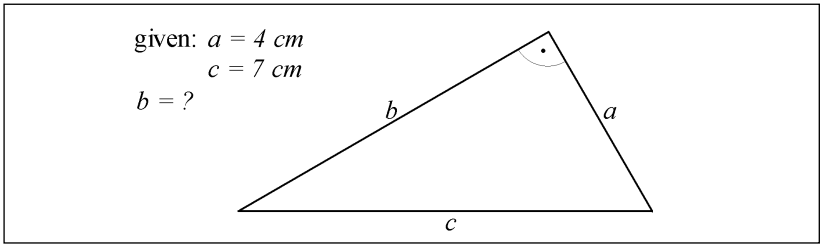


**Figure 3.** Example of an assessment problem for the knowledge domain "right triangles"

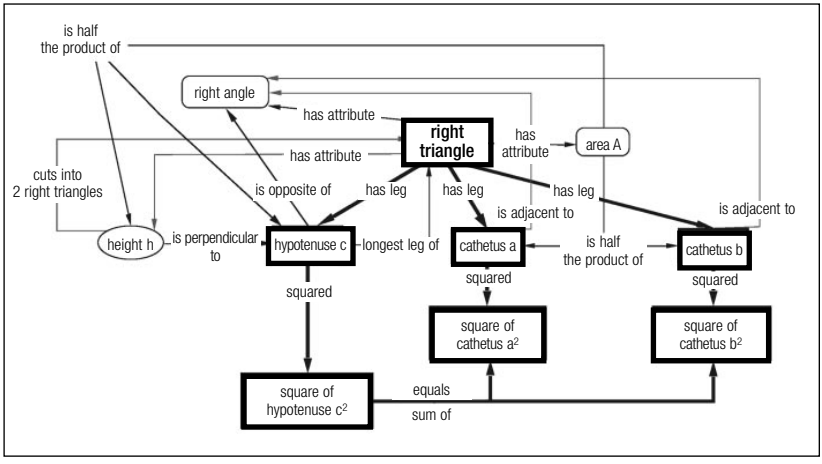


**Figure 4.** Concept map of the knowledge domain "right triangles." The marked substructure refers to the skill "knowing the Pythagorean Theorem."

cept map most probably will contain concepts on a higher level of abstraction, for example, Theorem of Pythagoras. This is in contrast to the more fine-grained concept map presented before, which also captures the definition or content of these general concepts.

Curriculum and content analysis not only reveal the basic concepts of a domain, but also the learning objectives that are related to these concepts. Learning objectives include required activities of the learner and may be captured by so-called *action verbs*. Action verbs (e.g., *state*, or *apply* a theorem) describe the observable student performance or behaviour and may be annotated to the nodes of the concept map representing the concepts that are to be taught. The information provided by the concept map then again can be used for establishing a competence structure in the sense of Knowledge Space Theory.

The concept map provides a hierarchical structure on the concepts of a domain. For instance, according to the curriculum the Pythagorean Theorem constitutes a prerequisite to the Altitude Theorem. This induces an order on the set of concepts $C$. The relation between the concepts may be represented graphically as in Figure 5(a). Additionally, a relation may be introduced on the set of action verbs $A$ that induces a structure on it. For instance, to "state" a particular theorem is most likely a prerequisite to "apply" the respective theorem, and therefore, the action verb "state" can be considered as a prerequisite to the action verb "apply." The structure defined on the action verbs can also be illustrated by a graph (see Figure 5(b) for an example).

Based on these considerations, a skill in terms of extended Knowledge Space Theory may be identified with a pair consisting of a concept and an action verb (e.g. $c_1a_2$). As an example for a skill consider "apply the Pythagorean Theorem," which consists of the concept "Pythagorean Theorem" and the action verb "apply." Formally we define the set of skills by $S \subseteq C \times A$ to reflect the fact that not all combinations of concepts and action verbs may be meaningful, or even realisable. A crucial question is how to merge the two kinds of structures, that is, the structure on the set of concepts and the structure on the set of action verbs, to establish a structure on the set of skills.

To resolve this issue we suggest the component-attribute approach (Albert & Held, 1994, 1999). According to this approach components are understood as dimensions, while attributes are the different values these dimensions can take on. In the present context, the set $C$ of concepts and the set $A$ of action verbs are considered as the components, and the attributes are identified with the respective elements (e.g. $c_1$, $c_2$, $c_3$, $c_4$ in $C$ and $a_1$, $a_2$ in $A$). On each component a relation is defined that orders the attributes (see Figure 5). A structure on the set of skills is then established by forming the direct product of these two components, which results in a prerequisite relation on the Cartesian product $C \times A$. The product of the two graphs displayed in Figure 5 is the relation depicted in Figure 6. From this you can see, e.g. that skill $c_2a_2$ is a prerequisite to the skills $c_2a_1$, $c_1a_1$, and $c_1a_2$, but to none of the other skills.

If $S$ is a proper subset of the Cartesian product $C \times A$ then we consider the prerequisite relation that the direct product shown in Figure 6 induces on $S$. In the framework of extended Knowledge Space Theory the prerequisite relation on the skills is interpreted as a surmise relation that gives rise to the competence structure. The competence states contained in it have to respect the ordering illustrated in Figure 6, which means, for example, that with the skill $c_3a_1$ each competence state has to contain the skills $c_3a_2$, $c_4a_1$, and $c_4a_2$, too.

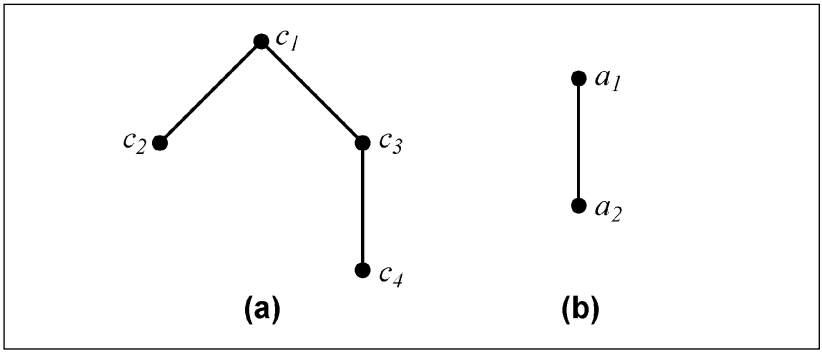


**Figure 5.** Concept structure (a) and structure defined on action verbs (b)

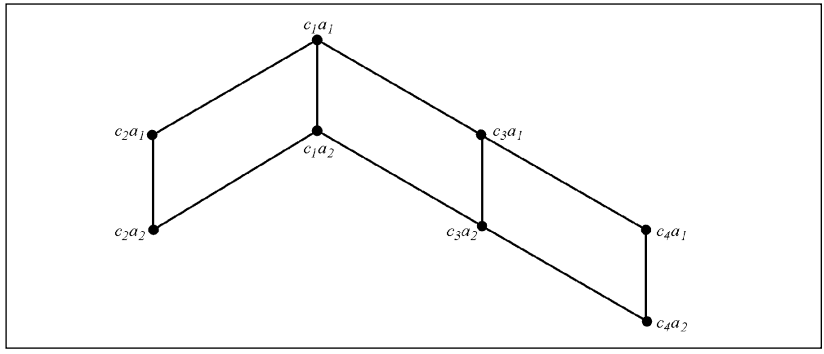


**Figure 6.** Example of a prerequisite relation on the skills induced by the structures on concepts and action verbs displayed in Figure 5.

Note, that from a psychological point of view, pairs consisting of a concept and an action verb, like "state Pythagorean Theorem" or "apply Pythagorean Theorem," describe rather global skills. Applying the Pythagorean Theorem might require several more elementary skills, which are in correspondence with the distinct steps in a solution path (e.g., extracting a root, transforming). It may thus be necessary to characterise the skills at a more fine-grained level. Further research is needed to decide upon an optimal level of granularity of the skills.

### Assigning Skills to Assessment Problems

Let us now consider the assignment of skills to the set of assessment problems. The relationship between assessment problems and skills can be formalised by two mappings.

- The mapping $s$ (skill function) associates to each problem a collection of subsets of skills. Each of these subsets (i.e., each competency) consists of those skills that are sufficient for solving the problem. Assigning more than one competency to a problem takes care of the fact that there may be more than one way to solve it.
- The mapping $p$ (problem function) associates to each subset of skills the set of problems that can be solved in it. It defines a knowledge structure because the associated subsets actually are nothing else but the possible knowledge states.

It has been shown that both notions are equivalent (Düntsch & Gediga, 1995), which means that, given the skill function, the problem function is uniquely determined, and vice versa. Consequently, only one of the two functions needs to be known to build the respective knowledge structure. Consideration is confined to the skill function, because it may be interpreted as representing the assignment of metadata to the problems. It follows

that assigning (semantic) metadata to assessment problems puts constraints on the possible knowledge states that can occur.

We illustrate the intimate relationship between skill function and problem function by a simple example. Consider the knowledge domain $Q = \{a, b, c, d\}$, and let the skill function $s$ on the set $S = \{x, y, z\}$ of skills be given by

$$s(a) = \{\{x, y\},\{x, z\}\},\ s(b) = \{\{x, z\}\},\ s(c) = \{\{x\},\{y\}\},\ s(d) = \{\{y, z\}\}.$$

This means, for example, that each of the skill sets $\{x, y\}$ and $\{x, z\}$ is sufficient for solving problem $a$. From the skill function we can derive the corresponding problem function, which yields

$$p(\emptyset) = \emptyset,\ p(\{x\}) = \{c\},\ p(\{y\}) = \{c\},\ p(\{z\}) = \emptyset,$$

$$p(\{x,y\}) = \{a, c\},\ p(\{x,z\}) = \{a, b, c\},\ p(\{y,z\}) = \{c, d\},\ p(S) = Q.$$

The assignment of skills to the assessment problems induces a knowledge structure on the set of problems, which is actually given by the subsets of problems in the range of the problem function. The knowledge structure for these examples is given by $\{\emptyset, \{c\}, \{a, c\}, \{c, d\}, \{a, b, c\}, Q\}$. Whenever a competence structure is available, e.g. as a result of exploiting ontological information (see "Deriving Skills and their Structure from Domain Ontologies"), the domain of the problem function is restricted to the actually occurring competence states. This puts additional constraints on the set of possible knowledge states.

In principle, the skill function for a given set $Q$ of assessment problems may introduce dependencies between skills, too. It may be the case that a certain skill is required for solving a problem only in connection with another skill. In the above example the skill $z$ is available only if either $x$ or $y$ is available. These dependencies, however, may only crop up in the given set $Q$, and it remains unclear whether they are valid in general. If capitalising on incidental dependencies between problems is to be avoided then the constraints the skill function puts on the possible subsets of skills should be neglected.

## Problem-Based Skill Assessment

A knowledge structure can form the basis for devising an efficient adaptive procedure for knowledge assessment (Doignon & Falmagne, 1999; Dowling & Hockemeyer, 2001). Problem-based skill assessment proceeds in two steps. First, the knowledge state of a learner, which refers to the observable behaviour, is adaptively assessed. After identifying a learner's knowledge state, the knowledge state can be mapped to the corresponding competence state in a second step.

Considering the knowledge structure given in Figure 2 for the knowledge domain $Q = \{a, b, c, d, e\}$, in the beginning of an assessment phase all states of the structure may correspond to the knowledge state of an individual learn-

er. According to a deterministic procedure, the assessment starts by selecting a problem that is contained approximately in half of the states of this structure and by posing this problem to the learner. Dependent on the learner's answer, the next problem will be selected. If the learner is capable of solving problem *b*, for example, then only the knowledge states containing problem *b* are still feasible. If subsequently problem *e* is solved, states {*a, b, c, e*} and {*a, b, c, d, e*} remain. The learner's knowledge state is uniquely identified after presenting problem *d*. For instance, state {*a, b, c, e*} results if problem *d* cannot be solved by this learner. Thus, for a set of five assessment problems, the presentation of only three problems allows for identifying the knowledge state of a learner. Formally, the number of questions for determining the knowledge state of a learner is approximately the dual logarithm of the total number of knowledge states.

Aside from the outlined deterministic assessment procedure, assessment may also be embedded into a probabilistic framework. A probabilistic assessment method allows for considering that the knowledge states may occur with different frequencies within a population as well as that a subject sometimes may be careless in answering a problem or may guess the correct answer. Such an assessment method assumes an a priori likelihood function (e.g. probability distribution) on the knowledge states. Initially, this likelihood may depend on the learner's profile, for example, the age, or grade of this learner. Later, this probability distribution is updated consistent with the learner's answers to the posed problems. The questioning continues until there is a pronounced peak in the likelihood function that suggests a unique knowledge state for an individual learner.

The knowledge state identified for a learner then can be mapped to his/her competence state by using the skill function. This means that, given a knowledge state, we are looking for the subset of skills that are sufficient for solving the problems contained in the knowledge state. However, there may be more than one such subset. In this case the skills cannot be recovered uniquely given the assessed knowledge state. To provide an example, consider the skill function defined in "Assigning Skills to Assessment Problems." If we assume that the assessment converged to the knowledge state {*c*} then it is unclear, which skills the learner is endowed with. According to the skill function either skill *x* or skill *y* may be responsible for solving problem *c*. This nonuniqueness occurs whenever a problem function is not one-to-one. Using additional information may lead to a unique identification of the available skills (e.g. looking up the learning history, checking for the skills actually taught). The best strategy, however, would be to select a proper set of assessment problems that avoids the nonuniqueness. Once the competence state of a learner has been determined it may serve as a basis for selecting a personalised learning path.

### Assigning Skills to Learning Objects

The relationship between learning objects and skills is different from that between assessment problems and skills. The relationship between the set $L$ of LOs and the skills in $S$ is mediated by two mappings (Hockemeyer, 2003; Hockemeyer et al. 2003). The mapping $r$ associates to each LO a subset of skills (required skills), which characterise the prerequisites for dealing with it, or understanding it. The mapping $t$ associates to each LO a subset of skills (taught skills), which refer to the content actually taught by the LO. In a similar way as previously outlined, the mappings $r$ and $t$ induce a learning structure on the set of LOs, which plays a central role for generating personalised learning paths. The pair of mappings $r$ and $t$ also imposes constraints on the competence states that can occur. Again, these constraints are tied to the given set $L$ of LOs. The imposed competence structure characterises the learning progress that may be achieved by studying the learning objects in $L$.

Generally, the assignment of skills to learning objects allows for deciding upon which learning objects are to be presented next, given a certain competence state. The concepts of inner and outer fringes (see "Basic Notions of Knowledge Space Theory") of a competence state may provide the basis for implementing personalised learning. The inner fringe of a competence state may be interpreted as "*what a learner can do*," while the outer fringe represents "*what this learner is ready to learn*." Therefore, proceeding in the learning process the next skills to be learned should be chosen from the outer fringe of the current competence state. Thus, a suitable learning object has to be selected that is characterized by required skills that the learner has already available and by taught skills that correspond to the outer fringe of the current competence state. If previously learned material has to be reviewed, then the content corresponding to the inner fringe of a learner's actual competence state seems to be a natural choice, because it contains the most sophisticated skills acquired by the learner.

## CONCLUSIONS

The present article proposes a competence-based extension of Knowledge Space Theory that provides a formal framework for explicitly linking assessment problems and learning objects to the relevant skills and competencies. It is demonstrated that the assignment of skills to assessment problems (which are sufficient for their solution) induces a knowledge structure characterising the possible answer patterns of the learners. Moreover, it is shown that assigning required and taught skills to learning objects allows for generating personalised learning paths. Introducing skills provides a general framework for relating models of the domain, the learners, and the learning objects, as described by Bouzeghoub, Defude, Duitama, and Lecocq (2006, this issue). These authors also refer to information about what is

required and what is provided by a LO, which is perfectly in line with the assignment of required and taught skills to LOs as discussed in "Assigning Skills to Learning Objects." The proposed skill assignments also contribute to the reusability of LOs (see Strijker & Collis, 2006, this issue).

The article provides a detailed discussion of how to derive relevant skills and their structure from domain ontologies. Two possible approaches are outlined. On the one hand, skills are identified with substructures of a concept map. On the other hand, skills are identified with pairs of concepts and action verbs, and a skill structure is established by merging the structures given on both sets. Assigning these skills to assessment problems and LOs, as suggested by the competence-based extension of Knowledge Space Theory, yields a framework for an efficient adaptive assessment of the skills and competencies of a learner, and for selecting personalised learning paths. This framework constitutes a valuable model for implementing personalised learning within an open technology-enhanced learning system. The implementation of the outlined theoretical framework within the iClass project is discussed by Türker, Görgün, and Conlan (2006, this issue), while Brady, O'Keeffe, Conlan, and Wade (2006, this issue) focus on the personalisation of the presented learning material via skill- or concept-based services offered by the Selector and the LO Generator module of the iClass system. A discussion of how to handle and integrate multiple skill assignments that characterize (partially overlapping) learning material coming from distributed resources is contained in Heller, Mayer, Hockemeyer, and Albert (2005).

## References

Albert, D., & Held, T. (1994). Establishing knowledge spaces by systematical problem construction. In D. Albert (Ed.), *Knowledge structures* (pp. 78–112). New York: Springer Verlag.

Albert, D., & Held, T. (1999). Component based knowledge spaces in problem solving and inductive reasoning. In D. Albert & J. Lukas (Eds.), *Knowledge spaces: Theories, empirical research applications* (pp. 15–40). Mahwah, NJ: Lawrence Erlbaum.

Bouzeghoub, A., Defude, B., Duitama, J.F., & Lecocq, C. (2006). A knowledge-based approach to describe and adapt learning objects. *International Journal on E-Learning, 5*(1).

Doignon, J. (1994). Knowledge spaces and skill assignments. In G. H. Fischer & D. Laming (Eds.), *Contributions to mathematical psychology, psychometrics and methodology* (pp. 111–121). New York: Springer Verlag.

Doignon, J.P., & Falmagne, J.C. (1985). Spaces for the assessment of knowledge. *International Journal of Man-Machine Studies, 23*, 175-196.

Doignon, J.P. & Falmagne, J.C. (1999). *Knowledge spaces*. Berlin: Springer.

Dowling, C. E., & Hockemeyer, C. (2001). Automata for the assessment of knowledge. *IEEE Transactions on Knowledge and Data Engineering, 13*, 451–461.

Düntsch, I., & Gediga, G. (1995). Skills and knowledge structures. *British Journal of Mathematical and Statistical Psychology, 48*, 9–27.

Falmange, J.C., Koppen, M., Villano, M., Doignon, J.P. & Johannesen, L. (1990). Introduction to knowledge spaces: How to build, test, and search them. *Psychological Review, 97*, 201-224.

Heller, J., Mayer, B., Hockemeyer, C., & Albert, D. (2005). Competence-based knowledge structures for personalised learning: Distributed resources and virtual experiments. In G. Albano, P. Ritrovato, & S. Salerno (Eds.), *Proceedings of the 1st International EleGI Conference on Advanced Technology for Enhanced Learning.* Swindon Wiltshire, UK: The British Computer Society (BCS).

Hockemeyer, C. (2003). Competence based adaptive e-learning in dynamic domains. In F. W. Hesse & Y. Tamura (Ed.), *The Joint Workshop of Cognition and Learning through Media-Communication for Advanced E-Learning (JWCL), 2003*, 79–82, Berlin.

Hockemeyer, C., Conlan, O., Wade, V., & Albert, D. (2003). Applying competence prerequisite structures for eLearning and skill management. *Journal of Universal Computer Science, 9*, 1428-1436.

Korossy, K. (1997). Extending the theory of knowledge spaces: A competence-performance approach. *Zeitschrift für Psychologie, 205*, 53-82.

Korossy, K. (1999). Modeling knowledge as competence and performance. In D. Albert & J. Lukas (Eds.), *Knowledge spaces: Theories, empirical research, applications* (pp. 103-132). Mahwah, NJ: Lawrence Erlbaum.

O'Keeffe, I., Brady, A., Conlan, O., & Wade, V. (2006). Just-in-time generation of pedagogically sound, context sensitive personalized learning experiences. *International Journal on E-Learning, 5*(1).

Strijker, A., & Collis, B. (2006). Strategies for reuse of learning objects: Context dimensions. *International Journal on E-Learning, 5*(1).

Türker, A., Görgün, I., & Conlan, O. (2006). The challenge of content creation to facilitate personalized eLearning experiences. *International Journal on E-Learning, 5*(1).

Zaluski, A. (2001). Knowledge spaces mathematica package. In *Proceedings of PrimMath 2001 - Mathematica in Science, Technology and Education*. Zagreb, Croatia, University of Zagreb.

## Acknowledgement

The work presented in this paper is partially supported by the European Community under the Information Society Technologies (IST) program of the 6th FP for RTD – project iClass contract IST-507922. The authors are solely responsible for the content of this paper. It does not represent the opinion of the European Community, and the European Community is not responsible for any use that might be made of data appearing therein.